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MATDIP301

Third Semester B.E. Degree Examination, Aug./Sept.2020
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If n is positive integer prove that :
$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$. **(06 Marks)**

b. Sum the series : $1 + x\cos\theta + \frac{x^2}{2!}\cos 2\theta + \frac{x^3}{3!}\cos 3\theta + ----+\infty$. **(07 Marks)**

c. Put the complex number $\left(\frac{2+i}{3-i}\right)^2$ into polar form. **(07 Marks)**

2 a. Find the n^{th} derivative $e^{ax}\cos(bx+c)$. **(07 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(2x+1)(x-2)}$. **(06 Marks)**

c. If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **(07 Marks)**

3 a. With usual notations prove that $\tan\varphi = \frac{rd\varphi}{dr}$. **(07 Marks)**

b. Find the pedal equation : $r^m = a^m \cos m\theta$. **(06 Marks)**

c. Expand $\log(1+\sin^2 x)$ in powers of x as for as the term in x^6. **(07 Marks)**

4 a. If $Z = e^{ax+by}f(ax-by)$ prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2ab\,z$. **(06 Marks)**

b. If $Z = f(x, y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **(07 Marks)**

c. If $u = \frac{xy}{z}, v = \frac{yz}{x}, w = \frac{zx}{y}$. Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. **(07 Marks)**

PART – B

5 a. Derive the reduction formula for $I_n = \int_0^{\pi/2} \cos^n x \, dx$. (07 Marks)

b. Evaluate $\int_0^1 x^5 \sin^{-1} x \, dx$. (06 Marks)

c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dx \, dy$. (07 Marks)

6 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$. (07 Marks)

b. Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta} \, d\theta$ in teams of gamma functions. (06 Marks)

c. Prove that $\beta(m, \tfrac{1}{2}) = 2^{2m-1} \beta(m, m)$. (07 Marks)

7 a. Solve $(x + y + 1)^2 \dfrac{dy}{dx} = 1$. (07 Marks)

b. Solve $(x \tan\dfrac{y}{x} - y\sec^2\dfrac{y}{x})dx + x\sec^2\dfrac{y}{x} dy = 0$. (06 Marks)

c. Solve $\dfrac{dy}{dx} - \dfrac{\tan y}{1+x} = (1+x)e^x \sec y$. (07 Marks)

8 a. Solve $\dfrac{d^4y}{dx^4} + y = \sin 2x \sin x$. (07 Marks)

b. Solve $\dfrac{d^2y}{dx^2} + 4y = x^4$. (06 Marks)

c. Solve $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$. (07 Marks)

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